N-63-4-3

FTD-TT- 63-485

TRANSLATION

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English Pages: 10

SOURCE: Russian Periodical, Izvestiya Akademii Nauk SSSR, Otdeleniye Tekhnicheskikh Nauk, Mekhanika i Mashinostroyeniye, 5, 1962, pp. 26-31

T-113
SOV/179-62-0-5-1-12

PREPARED BY:

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FLOW PAST A PLATE AT HIGH SUPERSONIC VELOCITIES

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In accordance with the analogy of an "explosion wave" the problem of high supersonic flow past bodies reduces to studying the gas movement in the transverse plane.

The problem of a flat explosion is equivalent to symmetrical flow past a flat blunt plate. The solution of this problem has a particularly simple form if we disregard the effect of the initial pressure (self-similar solution of the problem of a "strong explosion" [1]).

The problem of a flat explosion with subsequent movement of the piston corresponds to flow past a flat blunt plate at an angle of attack. In this case, even if we disregard the initial pressure the problem is significantly complicated, and its precise solution is possible by numerical methods.

The approximate solution of this problem can be carried out if it is limited to small angles of attack when blunting of the plate exerts the determining influence on the nature of the flow. Additional influence due to rotation of the flow on the plate (piston movement) is determined from the equation for a shock wave.

We note that an approximate solution to the analogous problem of flow past a blunt wedge was obtained by G. G. Chernyy [2] by an expansion of the flow parameters into series of powers of $(\gamma - 1)/(\gamma + 1)$, where γ is the ratio of specific

heats. The solution of the problem concerning flow past a blunt plate at an angle of attack by a method of small disturbances has been given [3].

1. We will introduce an equation for determining the position of a shock wave. For this we will use integral ratios of the law of conservation of energy, momentum, and mass, applying them to a volume of gas enclosed between the piston (surface of the body) and the shock wave.

If we assume that the width of the gas layer is equal to unity and disregard the initial pressure, the indicated ratios have the form:

$$\int_{r_0}^{R}\left(\frac{\rho u^2}{2}+\frac{p}{\gamma-1}\right)dr = E+\int_0^t p_0\dot{r}_0 dt, \quad p_0=\frac{d}{dt}\int_{r_0}^{R}\rho u dr, \quad \rho_1 R=\int_{r_0}^{R}\rho dr \qquad (1.1)$$

Here E is the energy released during the explosion;

ρ is the density;

p is the pressure;

u is the velocity of the transverse gas flow;

γ is the ratio of specific heat;

r_0 and R are the positions of the piston and shock wave at time t. We will indicate parameters on the piston (surface of the body) by the subscript 0, the undisturbed gas by the subscript 1, and the parameters immediately behind the shock wave by the subscript 2. The dot over r_0 and R represents differentiation with respect to t.

The solution of system (1.1) can be carried out by using some distribution of parameters in the transverse layer between the piston and the shock wave [4].

Let us now turn to the third equation of (1.1). It follows from this equation that regardless of what law of density distribution in the transverse layer $\rho(r)$ is taken, the magnitude of the

integral on the right side will remain constant. In particular, this will also be the case if the entire gas mass is concentrated about the shock wave [2,4].

In this limiting case the density distribution can be represented in the following form:

$$\rho(r) = \rho_1 R \delta(r - R)$$

where $\delta(r - R)$ is the Dirac function (delta-function).

After this it is not difficult to determine the following integrals from System (1.1)

$$\int_{r_0}^{R} \rho u dr \approx \rho_1 u_2 R, \qquad \int_{r_0}^{R} \frac{\rho u^2}{2} dr \approx \frac{\rho_1 u_2^2}{2} R \qquad \left(u_2 = \frac{2}{\gamma+1} \dot{R}\right)$$

where u_2 is the velocity behind the shock wave [1].

Disregarding the pressure change across the layer [2] we will also calculate the integral

$$\int_{r_0}^{R} \frac{p}{\gamma - 1} dr \approx \frac{p_0}{\gamma - 1}(R - r_0)$$

If we now substitute these approximate relations into the first and second equations of (1.1), we then obtain the pressure on the piston (surface of the body)

$$p_0 = \frac{2\rho_1}{\gamma+1}(\dot{R}^2 + R\ddot{R}) \tag{1.2}$$

the equation for determining the position of the shock wave

$$\frac{2\gamma}{\gamma+1} R\dot{R}^2 + R^2\ddot{R} - r_0(\dot{R}^2 + R\ddot{R}) = \frac{\gamma^2 - 1}{2}\frac{E}{\rho_1} + (\gamma - 1)\int_0^t \dot{r}_0(\dot{R}_2 + R\ddot{R})\, dt \tag{1.3}$$

In a case of a flow past a plane at an angle of attack, $r_0 = \alpha Vt$ (V is the incident-flow velocity; α is the angle of attack; for the upper surface the value α is taken with the minus sign) and Eq. (1.3) assumes the following form:

$$\frac{2\gamma}{\gamma+1} R\dot{R}^2 + R^2\ddot{R} - \alpha V(\dot{R}^2 + R\ddot{R})t = \frac{\gamma^2-1}{2}\frac{E}{\rho_1} + (\gamma-1)\alpha V R\dot{R} \qquad (1.4)$$

2. We will examine the particular cases of solving Eq. (1.4). After setting $\alpha = 0$ we obtain a solution of the problem of a "strong explosion" ($R = R_0$) which corresponds to symmetrical flow past a blunt plate

$$\frac{2\gamma}{\gamma+1} R_0\dot{R}_0^2 + R_0^2\ddot{R}_0 = \frac{\gamma^2-1}{2}\frac{E}{\rho_1}$$

From this,

$$R_0 = \left[\frac{9}{4}\frac{(\gamma+1)^2(\gamma-1)}{3\gamma-1}\right]^{1/3}\left(\frac{E}{\rho_1}\right)^{1/3} t^{2/3} \qquad (2.1)$$

and the pressure magnitude according to Eq. (1.2)

$$p_0 = \rho_1\left[\frac{4}{9}\frac{(\gamma+1)(\gamma-1)^2}{(3\gamma-1)^2}\right]^{1/3}\left(\frac{E}{\rho_1}\right)^{2/3} t^{-2/3} \qquad (2.2)$$

In order to now obtain flow past a flat blunt plate it is sufficient to substitute into Eqs. (2.1) and (2.2)

$$E = \frac{1}{2} C_x \frac{\rho_1 V^2}{2} d, \qquad t = \frac{x}{V}$$

where C_x is the blunt drag coefficient, d is the thickness of the plate; as a result we obtain

$$\frac{R_0}{d} = \left[\frac{9}{16}\frac{(\gamma+1)^2(\gamma-1)}{3\gamma-1}\right]^{1/3} c_x^{1/3}\left(\frac{x}{d}\right)^{2/3} \qquad (2.3)$$

$$\frac{P_0}{1/2\rho_1 V_1^2} = \left[\frac{2}{9}\frac{(\gamma+1)(\gamma-1)^2}{(3\gamma-1)^2}\right]^{1/3} c_x^{2/3}\left(\frac{x}{d}\right)^{-2/3} \qquad (2.4)$$

Formulas (2.3) and (2.4) coincide with the solutions obtained by G. G. Chernyy [2]. This same work gives a comparison with the calculations by a precise theory of a "strong explosion."

We can show that (2.1) is the solution of Eq. (1.4) not only when $\alpha = 0$ but as well as for any other $\alpha \neq 0$ if the limiting case $t \to 0$ is considered.

In another limiting case $t \to \infty$ the solution of Eq. (1.4) and the pressure magnitude have the following forms, respectively:

$$R = \frac{\gamma+1}{2}\alpha V t + \frac{\gamma-1}{\alpha^2 V^2}\frac{E}{p_1}, \qquad \frac{p_n}{{}^1/_2\rho_1 V} = (\gamma+1)\alpha^2 \tag{2.5}$$

The pressure magnitude determined by (2.5) coincides with the exact solution for a tapered plate.

Thus, if close to the leading edge of the blunt plate ($t = x/V \to \to 0$) the flow is determined mainly by blunting, then far downstream ($t \to \infty$) the effect of blunting diminishes and the pressure magnitude tends toward the value occurring at a tapered plate.

In order to obtain a solution to (1.4) for small but finite t, we assume that $R = R_0 + R_1$, where R_2 is defined (2.1) and R_1 is the addition due to rotation of the flow (piston movement). Here no assumptions are made relative to the magnitude of R_1; however the following conditions must be satisfied: 1) when $t \to 0$, $R_1 \to 0$ and the pressure magnitude should tend to a value (2.2); 2) the appearance of additional magnitude R_1 is connected only with the presence of an angle of attack (piston movement); in other words, parameter α should definitely enter into the solution of R_1.

If we now assume $R_1 \sim t^m$ and set $R = R_0 + R_1$ into Eqs. (1.2) and (1.4), we find that the first condition will be fulfilled when $m > 2/3$, and the second when $m > 5/6$.

Leaving in converted equation (1.4), i.e., after substitution of the relation $R = R_0 + R_1$ into it, the terms containing t to the lowest power (when $m > 5/6$ these will be the terms with $t^{1/3}$ and $t^{m-2/3}$),we obtain an approximate equation for determining R_1

$$t^2\ddot{R}_1 + \frac{8}{3}\frac{\gamma}{\gamma+1}t\dot{R}_1 + \frac{4}{9}\frac{\gamma-1}{\gamma+1}R_1 = \frac{2}{9}(3\gamma-2)\alpha V_1 t \tag{2.6}$$

The equation under consideration is an Euler equation with the right side [5].

The general solution of this equation has the following form:

$$R_1 = c_1 t^{m_1} + c_2 t^{m_2} + R_{11}$$

where c_1, c_2 are arbitrary constants, m_1, m_2 are roots of the characteristic equation, and R_{11} is the particular solution of (2.6).

Solution of Eq. (2.6) leads to negative values of m_1 and m_2 $(\gamma > 1)$ and, therefore, in order to satisfy the boundary conditions it is necessary to assume that $c_1 = c_2 = 0$.

Thus, only the particular solution of (2.6) remains. It is not difficult to see that it will be as follows:

$$R_{11} = \frac{(\gamma+1)(3\gamma-2)}{2(7\gamma-1)} \alpha V t$$

Finally, the solution of Eq. (1.4) for small values of $\underline{t}$ has the form

$$R = \left[\frac{9}{4} \frac{(\gamma+1)^2(\gamma-1)}{3\gamma-1}\right]^{1/3} \left(\frac{E}{\rho_1}\right)^{1/3} t^{2/3} + \frac{(\gamma+1)(3\gamma-2)}{2(7\gamma-1)} \alpha V t \qquad (2.7)$$

By substituting

$$E = \frac{1}{2} c_x \frac{\rho_1 V^2}{2} d, \qquad t = \frac{x}{V}$$

Formula (2.7) is reduced to form

$$\frac{2}{\alpha} z(R) = \left[\frac{9}{2} \frac{(\gamma+1)^2(\gamma-1)}{3\gamma-1}\right]^{1/3} [z(x)]^{2/3} + \frac{(\gamma+1)(3\gamma-2)}{2(7\gamma-1)} 2z(x) \qquad (2.8)$$

$$z(x) = \frac{\alpha^3}{c_x} \frac{x}{d}$$

It is completely obvious that the obtained solution of (2.7) or (2.8) is correct not only during flow past a blunt plate at an angle of attack, but also, e.g., during symmetrical flow past a blunt wedge.

Figure 1 shows the comparison of results of calculation by Formula (2.8) (dashed line) with the solution of G. G. Chernyy ($\gamma = 1.4$)

for a case of flow past a blunt wedge [2] (on the ordinate axis $\chi(R) = 2z(R)$.

Using (2.7) we determine by Formula (1.2) the pressure on the surface of the blunt body

$$\frac{T p_0}{\rho_1 v_1^2 \alpha^3} = f_1(\gamma) z^{-2/3} + f_2(\gamma) z^{-1/3} + f_3(\gamma) \tag{2.9}$$

Here

$$f_1(\gamma) = \left[\frac{(\gamma+1)(\gamma-1)^2}{36(3\gamma-1)^2}\right]^{1/3}, \qquad f_2(\gamma) = \frac{5}{3}\frac{3\gamma-2}{7\gamma-1}\left[\frac{(\gamma+1)^2(\gamma-1)}{6(3\gamma-1)}\right]^{1/3}.$$

$$f_3(\gamma) = \frac{(\gamma+1)(3\gamma-2)^2}{2(7\gamma-1)^2} \qquad \left(z = \frac{\alpha^3}{c_x}\frac{x}{d}\right)$$

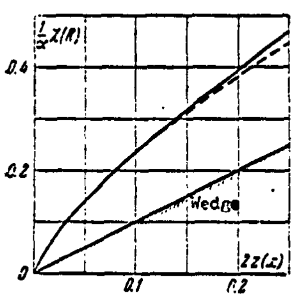

Fig. 1.

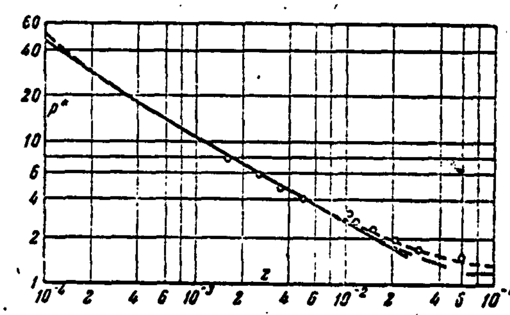

Fig. 2.

In particular, for $\gamma = 1.4$ the following result is obtained:

$$p^* = \frac{p_0}{\rho_1 V^2 \alpha^2} = 0.1 z^{-2/3} + 0.206 z^{-1/3} + 0.075 \tag{2.10}$$

For comparison let us introduce the formula which was obtained [3] by the method of small disturbances:

$$p^* = \frac{p_0}{\rho_1 V^2 \alpha^2} = 0.085 z^{-2/3} + 0.194 z^{-1/3} \tag{2.11}$$

Figure 2 shows the results of calculations by Formulas (2.10) (dashed line) and (2.11) (solid line). The small circles and the small dashes indicate Bertram's data which he obtained by the method

c. characteristics for $\alpha = 5^{\circ}$, 10°, with $M_1 = 20$ [3].

3. Let us turn to determination of the aerodynamic characteristics of a flat blunted plate with flow-past at high supersonic velocities at small angles of attack α.

In accord with Formula (2.9) the coefficients of relativ… at the lower and upper surfaces are determined by the following ratios:

$$p_+ = \frac{p_0}{1/2\rho_1 V^2} = 2f_1(\gamma)\, c_x^{2/3}\left(\frac{x}{d}\right)^{-2/3} + 2f_2(\gamma)\, c_x^{1/3}\left(\frac{x}{d}\right)^{-1/3}\alpha + 2f_3(\gamma)\,\alpha^2 \qquad (3.1)$$

$$p_- = \frac{p_0}{1/2\rho_1 V^2} = 2f_1(\gamma)\, c_x^{2/3}\left(\frac{x}{d}\right)^{-2/3} - 2f_2(\gamma)\, c_x^{1/3}\left(\frac{x}{d}\right)^{-1/3}\alpha + 2f_3(\gamma)\,\alpha^2 \qquad (3.2)$$

From (3.2) it follows that on the upper surface there possibly exists such a point 0, at which the pressure will be equal to zero.

From physical considerations this point can be examined as a point in which breakaway occurs. If we disregard the third term as a small magnitude compared with the other terms, from (3.2) we obtain

$$\left(\frac{x}{d}\right)_0 = \frac{9}{250}\,\frac{(\gamma-1)(7\gamma-1)^2}{(\gamma+1)(3\gamma-1)(3\gamma-2)^2}\,\frac{c_x}{\alpha^2} \qquad (3.3)$$

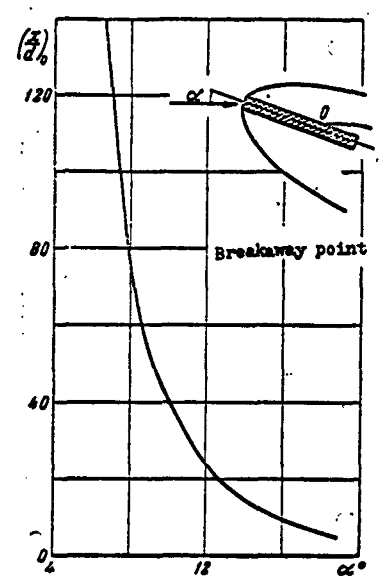

Fig. 3.

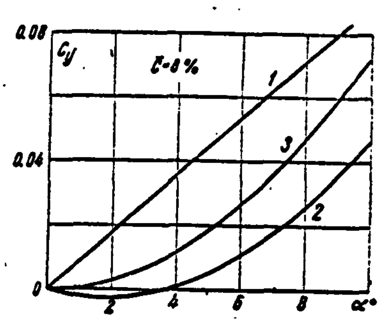

Fig. 4.

Figure 3 shows the dependence (3.3) when $\gamma = 1.4$ and $C_x = 1.84$ (flat nose). As is evident from Fig. 3, with an increase of the

angle of attack the breakaway point moves forward toward the leading edge of the plate. At large angles of attack the breakaway point is located on the leading edge, and the pressure on the whole upper surface will be equal to zero. A similar phenomenon occurs during flow past a plate according to the Newton system.

If we limit ourselves to an examination of such angles of attack for which point O is positioned behind the plate, then the lift coefficient is defined as:

$$c_y = \int_0^1 (p_+ - p_-)\, d\left(\frac{x}{d}\right) = 6 f_2(\gamma)\, c_x^{1/3} \left(\frac{d}{b}\right)^{1/3} \alpha$$

Here b is chord and d is the thickness of the plate.

We obtain the total lift coefficient if the pressure on the leading part of the plate is taken into consideration:

$$c_y = \left[6 f_2(\gamma)\, c_x^{1/3} \left(\frac{d}{b}\right)^{1/3} - c_x \frac{d}{b}\right] \alpha \qquad (3.4)$$

Figure 4 shows the dependence (3.4) constructed for $\gamma = 1.4$ and $C_x = 1.84$ (curve 1). For comparison, we have: the lift coefficient of a blunt plate obtained without taking into account the effect of blunting on flow past a plate (curve 2):

$$c_y = (\gamma + 1)\alpha^2 - c_x \alpha \frac{d}{b}$$

the left coefficient of a tapered plate (curve 3)

$$c_y = (\gamma + 1)\alpha^2$$

As was to be expected, at large angles of attack the effect due to blunting diminishes and the characteristic $c_y = f(\alpha)$ of a blunt plate approximates those values obtained for a tapered plate.

The coordinate of the center of pressure X on the flat blunt plate relative to the leading edge is defined by the following expression

$$\frac{X}{b} = \left(\int_0^1 (p_+ - p_-) \frac{x}{b} d \frac{x}{b}\right) \left(\int_0^1 (p_+ - p_-) d \frac{x}{b}\right)^{-1} = 0.4 \qquad (3.5)$$

Thus, due to blunting the pressure center of a plate shifts forward. With an increase of the angles of attack the pressure center will move into a position corresponding to a tapered plate.

Submitted February 8, 1962

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